16117

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions : (1) All Questions are *compulsory*.

(2) Illustrate your answers with neat sketches wherever necessary.

(3) Figures to the right indicate full marks.

(4) Assume suitable data, if necessary.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any TEN of the following : **20**

(a) Find x if $\begin{vmatrix} 4 & 3 & 9 \\ 3 & 2 & 7 \\ 1 & 4 & x \end{vmatrix} = 0$.

(b) If $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, find $2A + 3B - 5I$, where I is unit matrix of order two.

(c) If $A = \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$, show that A^2 is null matrix.

(d) Resolve into partial fraction: $\frac{1}{x(x+1)}$.

(e) Prove that $\cos 2\theta = 2\cos^2\theta - 1$.

(f) Find $\sin\alpha$ if $\tan\left(\frac{\alpha}{2}\right) = \frac{1}{\sqrt{3}}$.

(g) Without using calculator find the value of $\sin(-765°)$.

(h) Find the principal value of $\sec\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$.

(i) Define compound angle.

(j) Prove that the lines $3x + 2y = 5$ and $2x - 3y = 6$ are perpendicular.

(k) Find range and coefficient of range of following data :

50, 90, 120, 40, 180, 200, 80.

(l) Find AB if $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -5 \\ 0 & 1 \end{bmatrix}$

2. Attempt any FOUR of the following : **16**

(a) Solve the following equations using Cramer's Rule :

$2x + 3y = 5$; $y - 3z = -2$; $z + 3x = 4$

(b) If $A + I = \begin{bmatrix} 1 & 3 & 4 \\ -1 & 1 & 3 \\ -2 & -3 & 1 \end{bmatrix}$, obtain the matrix $(A + I)(A - I)$.

(c) Show that matrix $A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ is an orthogonal matrix.

(d) Find the inverse of the Matrix;

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ by adjoint method.

(e) Resolve into partial fraction : $\frac{x^3 + x}{x^2 - 4}$.

(f) Resolve into partial fraction : $\frac{3x - 1}{(x - 4)(2x + 1)(x - 1)}$.

3. Attempt any FOUR of the following : **16**

(a) Using Matrix inversion method solve the system of equations : $x + y + z = 3$, $3x - 2y + 3z = 4$, $5x + 5y + z = 11$.

(b) Resolve into partial fractions : $\frac{\tan\theta}{(\tan\theta + 2)(\tan\theta + 3)}$.

(c) Resolve into partial fraction :

$\frac{x^2 + 23x}{(x + 3)(x^2 + 1)}$.

(d) Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$.

(e) If $\tan A = \frac{1}{3}$, $\tan B = \frac{1}{4}$, where $0 < A < \pi/2$, $\pi < B < 3\pi/2$, find $\sin(A + B)$.

(f) Without using calculator, find the value of :

$\tan(585°) \cdot \cot(-495°) - \cot(405°) \cdot \tan(-495°)$

4. Attempt any FOUR of the following : **16**

(a) Prove that :

$\sin(A - B) = \sin A \cos B - \cos A \sin B$.

(b) Prove that : $\cos 2A = 2\cos^2 A - 1$.

(c) If $\tan(x + y) = \frac{1}{2}$ and $\tan(x - y) = \frac{1}{3}$, find (i) $\tan 2x$, (ii) $\tan 2y$.

(d) Prove that : $\frac{\sin A + \sin 2A + \sin 3A + \sin 4A}{\cos A + \cos 2A + \cos 3A + \cos 4A} = \tan\left(\frac{5A}{2}\right)$.

(e) Prove that : $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$.

(f) Prove that : $\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$.

5. Attempt any FOUR of the following : **16**

(a) Prove $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x + y}{1 - xy}\right)$ if $1 - xy > 0$.

(b) Prove that $\sin 10° \cdot \sin 30° \cdot \sin 50° \cdot \sin 70° = \frac{1}{16}$

(c) Prove that $\sin C - \sin D = 2\cos\left(\frac{C + D}{2}\right)\sin\left(\frac{C - D}{2}\right)$.

(d) Show that the distance between two parallel lines $ax + by + C_1 = 0$ &

$ax + by + C_2 = 0$ is given by $d = \left|\frac{C_2 - C_1}{\sqrt{a^2 + b^2}}\right|$.

(e) Find the length of perpendicular on the line $3x + 4y - 5 = 0$ from (3, 4).

(f) Find the equation of the line passing through the point of intersection of lines $2x + 3y = 13$, $5x - y - 7 = 0$ and perpendicular to the line $3x - 2y + 7 = 0$.

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6. Attempt any FOUR of the following : **16**

(a) Find the equation of line passing through the point of intersection of lines $x + y = 0$ and $2x - y = 9$ and a point (2, 5).

(b) Find the mean deviation from median of the following distribution :

Weight (in gms)	10-15	15-20	20-25	25-30	30-35	35-40	40-45
No. of items	7	12	16	25	19	15	6

(c) Calculate : (i) Standard deviation, (ii) Co-efficient of variation from the following data :

Rainfall	70-80	80-90	90-100	100-110	110-120	120-130	130-140	140-150
No. of places	06	07	12	19	21	18	11	06

(d) The weights of 100 students are given by the following distribution :

Weight above or equal to	36	41	46	51	56	61	66	71
No. of Students	100	96	79	56	28	11	5	2

Calculate : (i) Mean, (ii) Variance of the data using step deviation method. No student has weight above 75 kg.

(e) In the two factories A & B engaged in the same industry, the average weekly wages & standard deviation are as follows :

Factories	**Average wages**	**Standard deviation**
A	34. 5	5.0
B	28.5	4.5

Which factory is more consistent ?

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